# Transient Rendering

## an unexplored intersection of graphics, vision, and physics

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**Class presentation note:**

Please help clarify the language we use. Rename any idea you like. We know that, in particular, “transient” is not an obvious word for what we mean in this context.

# Outline

# Distinction

steady-state vs. transient light transport

# Steady-state Light Transport

- infinite speed of light
- videos are sequences of images of different static worlds
- fundamentally: we are reasoning about energy, number of photons, or irradiance at a pixel

# Transient Light Transport

- finite speed of light
- scattering in scene creates many paths
- each path takes different time to traverse
- even a single pulse can evolve into a complicated pattern in time
- fundamentally: we are reasoning about power, rate of incoming photons, or irradiant flux at a pixel

“transient” $\rightsquigarrow$ lasting only a short time, steady-state behaviour is usually dominant

# Motivation

Q: Humans can't see the effects so why should it matter?

A: There is growing interesting in time-of-flight based vision applications and we want some general, physical explanation of measurements we make.

This is our central motivation.

# Contributions

- transient rendering equation
- approximation procedure
- transient photometric response function
- sensor model

# Outline

1. Introduction
2. Background
   - SONAR
   - LIDAR
   - Rendering Equation
3. Transient Rendering Equation
4. Approximation Procedure
5. Transient Photometric Response Function
6. Sensor Model
7. Future Research Directions
8. Conclusion

# SONAR

(SOund NAvigation and Ranging, aka SODAR)

- speed of light is about a million times the speed of sound
- previous reseach advanced intricate models of the effects of many surfaces with complicated scattering properties
- yields ability to recover detailed models of the world from samples taken over time
- models are specific to sound (pervasive diffraction, no ray devices)
- Russell et al have produced a analysis pipeline including models of emission, propagation, scattering, media effects, complex sensors, and data interpretation in the context of 3D sea floor modelling [**?**]

Tie-in: applications are powerful and analysis is thorough but it does not describe light

# LIDAR

(LIght Detecting And Ranging, aka Laser Radar [?])

- over long distances the speed of light becomes noticeable
- sensors primarily report depth or distribution of depths
- most LIDAR models are extremely simple (single scatter, reflection only)

Tie-in: applications are not as powerful as sonar, analysis is much simpler, but it does deal with light in the short-term

# Rendering Equation

(referring to the rendering equation due to Jim Kajiya in 1986 [**?**])

- in graphics, historically, many model of light transport existed for specific, idealized worlds
- rendering equation gave a physical explanation for observed light
- assumes a bidirectional scattering function (BSDF), infinte speed of light, and lesser some details
- suggested new way of computing images
- usually referenced as either vacuum or volume variant
- most rendering methods can be seen as approximations to the rendering equation (or slight generalizaitons of it)

# Rendering Equation

- operator form: $R = R_0 + GR$ ($R$ is radiance, $R_0$ is emitted light, $G$ is global light transport)
- can be pretty in pretty integral form (but too pretty for this talk)
- note: the definition of $R$ depends on $R$
- note: $G$ traditionally includes a geometry term and a visibility term

Tie-in: gives a common story to a huge space of rendering applications but does not tell us about short-term effects due to propagation delay

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## Statement

The radiant flux at a point in space at a point, in time, in a direction is the light that originates at that point, at that time, in that direction plus the light that scatters through that point, at that time, in that direction from nearby points, delayed by propagation time (assuming outbound flux is only a funciton of inbound flux, summarized by a BSDF kernel).

OR

$$R = R_0 + GR$$

with $R$ read as radiant flux, a function of time (familiar???)

# Differences from Traditional Rendering Equation

- finite instead of infinite speed of light
- power instead of energy

(the rest is the same!)

# Decomposing $G$

Global light transport $G$ is the composition of two physical processes:

- propagation, $P$
  - turns radiant flux into irradiant flux
  - light at one point arrives at another delayed by a time proportional to the distance between them
- scattering, $S$
  - turns irradiant flux into radiant flux
  - happens at a point so no delay is incurred

$R = R_0 + SPR$ where $P$ is taken to include geometry and visiblity terms and $S$ is analogous to traditional scattering but defined over flux functions

# Interpretation

Q: What does it tell us?

A: It tells is the power of light

- at every point!
- in every direction!
- at every time!

As expected, this is exactly “global illumination” in terms of flux. It has a pretty integral form, but we need to commit to additional details about the world to write it out.

# Discussion

- we now have some physically-motivated definition for $R$ for quite arbitrary worlds
- form echos structure of traditional rendering equation (both derived from physics)
- can be specialized into transient vacuum and volume rendering equations given some description of the world

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# Motivation

The transient rendering equation is recursive!

We need a formulation that defines $R$ only in terms of givens.

# Derivation

$$
\begin{aligned}
R &= R_0 + GR && \text{transient rendering equation} \\
R - GR &= R_0 && \text{subtract move G R} \\
(I - G)R &= R_0 && \text{factor out operators, } I \text{ is identity} \\
R &= (I - G)^{-1} R_0 && \text{apply inverse} \\
R &= (G^0 + G^1 + G^2 + \cdots) R_0 && \text{Neumann series} \\
R &= R_0 + GR_0 + G^2 R_0 + \cdots && \text{distribute}
\end{aligned}
$$

The final result is an exact definition for $R$ using only $G$ and $R_0$.
If $G$ ($S$ in particular) consumes power quickly, then just a few terms of the sum will suffice to produce an accurate approximation of $R$.

# World Model

- collection of interfaces (flat surfaces) with geometry and BSDF
- mostly free space where light propagates freely at some speed
- point light sources with defined radiant flux functions

# Notation

| | |
|---|---|
| $X$ | some point |
| $n$ | some direction |
| $t$ | sime time |
| $R_k(X, n, t)$ | radiant flux at after $k$ scattering events |
| $I_k(X, n, t)$ | irradiant flux after $k$ scattering events |
| $R$ and $I$ | $\sum_{k\geq 0} R_k$ and $\sum_{k\geq 0} I_k$ |
| $R_0$ and $I_0$ | either given or easily computable from point lights |
| $K(X, n, n')$ | scattering kernel defined on interfaces |
| $Y(X, n)$ | first intersection point |
| $G(X, Y)$ | geometry term |
| $D(X, Y)$ | propagation time for light from $Y$ to $X$ |

# Propagation and Scattering

Recall that $G = SP$.

- propagation, $P$
    - $k$-scattered irradiant flux is the sum of all visible $k$-scattered radiant flux, attenuated by the geometry term and delayed by propagation time
    - $I_k(X, n, t) = \int_{n'} G(X, Y) R_k(Y, n', t - D(X, Y)) d\omega$
- scattering, $S$
    - $k+1$-scattered radiant flux is the sum of $k$-scattered irradiant flux distributed by the scattering kernel
    - $R_{k+1}(X, n, t) = \int_{n'} K(X, n, n') I_k(X, n', t) d\omega$

# Procedure

1. tesselate the world by adding virtual interfaces so that light flies freely within each region
    - regions must be convex
    - virtual interfaces are “transparent” ($K = \delta(n - n')$)
    - boundary interfaces (those not touching another region) may absorb all light for simplicity ($K = 0$)
2. calculate $R_0$ and $I_0$ from known light sources
3. repeat until satisfied:
    1. calculate $R_k$ using $S$ and $I_k$
    2. calculate $I_{k+1}$ using $P$ and $R_k$
4. sum $R_k$ to get approximate $R$

# Discussion

- we can approximate $R$ to arbitrary accuracy
- adding virtual interfaces inflates the scattering index (requiring more terms) but ensures visibility term is always 1, avoiding repeated shadow calculation
- keep in mind that $R$ is a function of a point, direction, and time and needs a suitable representation in practice

# Outline

# Motivation

The transient rendering equations tells us far more than we need to know!

We can imagine real, physical sensors with specific

- eye position $E$
- direction $n$
- input light with direct flux $R_0$

# Definition

The transient photometric response function is the flux at the eye point, opposite the eye direction, as a function of time resulting from a pulse of light at given light point.

$$f(t) = I(E, -n, t) = R(Y, -n, t - D(E, Y))$$

What's in a name?

- "transient" from transient rendering equation
- "photometric" for light as opposed to general radiation
- "response" from results of light impulse
- "function" from being a function of time

# Discussion

- the TPRF is (almost) directly sensed in many LIDAR applications, but only simple properties of it are examined (ex: time to first peak)
- the TPRF model is highly idealized but has important properties to be seen in the next section

# Outline

# Motivation

In reality

- sensors report function of time as a sequence of samples
- sensors must integrate over time to build samples (with some time-sensitivity envelope)
- lights cannot pulse for infinitely short periods (have some amplitude envelope in time)
- light comes in photons
- real devices have internal noise sources

# Composition

We address the realities by processing the TPRF, leveraging the definition in terms of a single point light source with impulse emission pattern.

- light envelope: convolve TPRF with envelope
- sensitivity envelope: multiply TPRF with envelope
- sampling: integrate over (all) time
- photons: use reasonable scenes (not to small, not too dim)
- noise: add a generic AGWN term

$$\text{Measurement}_i = \text{Noise} + \int_t (\text{TPRF}(t) * \text{Light}(t))\, \text{Sensitivity}_i(t) dt$$

# Discussion

- have a very general sensor model with physical motivation
- TPRF encapsulates all of the world-dependent parts of the model

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# Generalizing the Model

- sub-surface scattering
    - move directly to transient volume rendering
    - generate appropriate geometric detail and use our procedure
- phosphoresence
    - our derivation assumes interfaces cannot release energy they have stored
    - allow excitation at each point on interfaces subject to some model
- wavelength
    - take dispersion into account at interfaces
    - allow different wavelength for emission and absorption in phosphoresence

# Implementing the Model

- dependency calculation
    - our approximation procedure limited light transport to regions
    - presumably, some procedure exists for telling exactly how many passes are needed in which regions to calculate a TPRF to the desired accuracy
- function representations
    - try analytic expressions
    - try point samples (photons at specific points, directions, times)
- augment a common raytracer
    - may be difficult because of fundamental energy vs. power assumptions

# Building a Sensor

- modify existing LIDAR system
- string together a photosensor, LED, and really expensive oscilloscope

This sounds easy enough!

# Applications

- uncover hidden assumptions in traditional 2.5-D range finding
- attempt 3.0-D range finding (shape recovery include hidden surfaces)
- recover sub-surfaces scattering parameters from time instead of space
- blindly decompose TPRF into individual bounce layers
- incorporate information only visible on transient scale into traditional video displays (augmented human vision???)

# Outline

# Conclusion

- taken initial steps into exploring effects of propagation delay for light in account, called this transient rendering
- defined a physically-relevant generalization of the rendering equation
- defined a method for approximating the rendering equation in terms of parameters for fairly general worlds
- defined a summary measure of transient light patterns
- defined a physically-motivated, general sensor model
- proposed a wide array of new research directions include previously unreachable applications

We hope that transient rendering can serve as a principled foundation for future time-of-flight-based computer vision techniques.